Note on the Position of the Sun's Axis of Rotation, as deduced from Greenwich Sun-spot Measures 1886–1901. *Papers of the I.U.S.R. Computing Bureau, No. I.* By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. The present note is to be regarded rather as a statement of problems than as a solution of them. A discussion of the Greenwich sun-spot measures was begun a few years ago, as a necessary preliminary to computations in connection with the International Union for Solar Research; and it is being continued as the work of the Computing Bureau, of which the writer was asked to take charge. But the resources of this Bureau are at present very slender, and the time of the Director is much occupied with the printing of the Astrographic Catalogue, so that it may be some time before a definitive discussion is completed. Meanwhile the following provisional statement may be of interest; it has been drawn up in response to one or two inquiries for information. Another note will shortly be presented, dealing with the velocity of rotation.

2. The material used is taken from the Greenwich sun-spot ledgers, 1886–1901 inclusive. Later publications, especially the welcome volume of "photoheliographic results, 1874–1885," which has just been issued, will allow of a considerable extension of the discussion; but these were not available when the following computations were commenced. The discussion was confined to spot groups which were seen for a period of ten days at least; and no notice is taken in what follows of the identity of recurring groups. The longitude and latitude on each day were compared with the mean values given in the ledgers; and the differences were tabulated under each 10° of distance from central meridian, so as to show the motions in latitude and longitude. The spots were further grouped according to their latitudes, in groups 0° to 10°, 10° to 15°, 15° to 20°, 20° to 25°, and over 25°, N. and S. latitudes being kept separate.

3. When we have a set of mean residuals for each 10° of longitude, various methods may be proposed for deducing the mean drift from the series. Take, for instance, the following series of mean residuals in latitude for spots in N. latitudes 0° to 10° in the month of October (all years combined).

Dist. from Central Merid.	Mean Resid.		Dist. from Central Merid.	Mean Resid.	
-90° to -80°	$+0^\circ{\cdot}50$		$+90^\circ$ to $+80^\circ$	$\overset{\circ}{—}$	
-80 to -70	$+\ {\cdot}28$		$+80$ to $+70$	$-0{\cdot}38$	
-70 to -60	$+\ {\cdot}73$	$+0^\circ{\cdot}40$	$+70$ to $+60$	$-\ {\cdot}32$	$-0^\circ{\cdot}38$
-60 to -50	$+\ {\cdot}20$		$+60$ to $+50$	$-\ {\cdot}44$	
-50 to -40	$+\ {\cdot}27$		$+50$ to $+40$	$-\ {\cdot}33$	
-40 to -30	$+\ {\cdot}23$	$+\ {\cdot}22$	$+40$ to $+30$	$-\ {\cdot}04$	$-\ {\cdot}24$
-30 to -20	$+\ {\cdot}16$		$+30$ to $+20$	$-\ {\cdot}34$	
-20 to -10	$+\ {\cdot}15$	$+\ {\cdot}06$	$+20$ to $+10$	$-\ {\cdot}36$	$-\ {\cdot}24$
-10 to 0	$-\ {\cdot}04$		$+10$ to 0	$-\ {\cdot}12$	

Differences of method would turn chiefly upon the relative weights to be assigned to the outer and inner residuals. The outer give better intervals for measuring the motion; but, on the other hand, they are affected by larger errors, owing to foreshortening, and possibly to optical distortion. We shall do no harm by making two separate determinations of drift from outer and inner groups and comparing the results. If we reject the outermost groups (80° to 90°) as too much foreshortened, and take the mean of the next three, we get results for $+65^\circ$ and -65°, a difference of 130° in longitude. The next three groups will give us mean values for $+35^\circ$ and -35°, a difference of 70°.

4. A drift in longitude may be taken to increase steadily with the time, and hence the differences for $\pm 65^\circ$ should be to those for $\pm 35^\circ$ in the ratio of 13 to 7. Thus, if the above figures had related to longitudes, the mean drift per 10° would have been $(-0^\circ{\cdot}38 - 0^\circ{\cdot}40)/13 = -0^\circ{\cdot}060$ from the outer groups and $(-0^\circ{\cdot}24 - 0^\circ{\cdot}22)/7 = -0^\circ{\cdot}066$ from the inner groups.

5. The same might be true of a drift in latitude if it were a physical drift; but if it is an apparent drift, due to a faulty determination of the Sun's axis, these formulæ will not apply. The error due to a wrongly assumed axis is of the form $k \sin(\theta + 65^\circ) - k \sin(\theta - 65^\circ) = 2k \cos\theta \sin 65^\circ$ in one case, and (similarly) $2k \cos\theta \sin 35^\circ$ in the other. The ratio is thus $\sin 65^\circ/\sin 35^\circ = 1{\cdot}58$ instead of $65/35 = 1{\cdot}86$.

6. To take the latitude first: if we have adopted a wrong axis, the effect will be a spurious latitude drift, varying in amount as we go round the Sun during the year. The drift will in fact be, roughly, similar to the Sun's drift in declination, if for a moment we regard the equator as an erroneous determination of the ecliptic: it will be northwards at one time of year and southwards six months later, with no drift midway. Hence, if we arrange the results according to the time of year, they will form a cycle. The results for latitude were accordingly collected in months, each spot being assigned to the calendar month in which it appeared on the central meridian; and the mean drifts between the meridians above specified are given in Tables I. and II., which require no further explanation.

TABLE I.

Latitude drift between +65° *and* −65° *in each month* (*of the years* 1886–1901).

Latitude.	−25°	−20°	−15°	−10°	0°	+10°	+15°	+20°	+25°	
Jan.	+78	−94	−77	−25	−23	−85	−87	−33	−32	−88
Feb.	+71	−67	−13	−7	−29	−7	+1	+30	−118	—
Mar.	−67	−15	−20	−39	−37	−43	−33	−76	+25	+186
Apr.	−28	—	+17	+14	−4	−19	−3	+22	+13	—
May	+15	+76	−54	+31	+77	−5	0	+2	−29	—
June	—	+5	−54	−17	+74	−20	−17	−2	+33	+120
July	+115	−18	+42	−75	+18	−19	−25	+93	−145	+270
Aug.	−43	−27	−12	−37	+6	+90	−41	−59	+16	−25
Sept.	+115	−29	−53	−13	−28	−33	−43	+13	+50	−12
Oct.	−113	−115	−7	−16	−3	−78	−75	−115	−188	−40
Nov.	−15	−159	−52	+33	−41	−59	−124	−116	−148	
Dec.	−82	−66	−57	−112	−13	−57	−119	−29	+70	+180
Mean	—	—	−28	−22	0	−28	−47	−22	—	—

The unit in the table is 0°·01, and the figures are differences between the mean residual at +65° and that at −65°. The row of figures at the top represents solar latitude.

TABLE II.

Latitude drift between +35° *and* −35° *in each month* (*of the years* 1886–1901).

Latitude.	−25°	−20°	−15°	−10°	0°	+10°	+15°	+20°	+25°	
Jan.	+30	+5	+20	−18	−42	−39	−48	−25	+87	−5
Feb.	+32	−30	−8	−16	−6	−26	+12	+22	−65	+25
Mar.	−35	−3	−9	−26	−19	−23	−20	−28	+30	+54
Apr.	−22	+107	+3	+9	+20	−15	−4	−15	+3	—
May	+28	−21	−50	+5	+22	+34	+25	+45	−42	—
June	−85	−27	−25	−30	+58	+4	−6	+6	+49	+20
July	+25	+15	+38	−6	−5	−29	−18	+50	−101	+165
Aug.	+22	−25	−12	−57	−6	+30	−39	−25	+15	+35
Sept.	+118	−5	−40	−9	+28	−1	−10	−37	+33	+32
Oct.	+50	−30	−55	−5	+7	−46	−38	−19	−193	+10
Nov.	+55	−54	−42	+53	−33	−16	−61	−86	−85	...
Dec.	−98	+10	−25	−57	−48	−67	−32	−29	−43	+125
Mean	—	—	−17	−13	−2	−16	−20	−12	—	—

Multiplying the coefficients A and B by $\frac{1}{2}$ cosec 65° for Table I., and $\frac{1}{2}$ cosec 35° for Table II., we get the error of the Sun's axis,

which we may now express (as Carrington does) in minutes of arc. The separate determinations are given in Table III.

Excluding, at any rate in the first instance, the outside groups (for latitudes above 20°), we can now analyse the remaining 6 columns in each table harmonically, to find an expression of the form—

$$A \sin \theta + B \cos \theta + C,$$

where $\theta = 0$ for the middle of January.

TABLE III.

Quantities representing the Errors of the Sun's adopted axis.

Latitudes.	−20°	−15°	−10°	0°	+10°	+15°	+20°	Mean.
A from Table I.	+2′·9	+3′·7	+6′·6	+4′·2	+14′·5	+12′·1		+7′·3
A ,, ,, II.	+7·1	−0·8	+6·5	+5·7	+12·1	+14·5		+7·5
B from Table I.	−5·8	−1·0	−11·6	−12·0	−9·5	−9·7		−8·3
B ,, ,, II.	+0·3	+1·3	−16·7	−13·7	−4·9	−10·6		−7·2

Inspection of Table III. suggests the following conclusions:—

(*a*) The determinations from meridians ±65° and ±35° are satisfactorily accordant. Thus the Sun's disc may be treated as free from distortion for the purpose in view.

(*b*) The adopted axis is sensibly in error for the period under discussion.

(*c*) The different zones give sensibly different values for the error, zones −20° to −15° and −15° to −10° being specially discordant. But the results seem really to fall into three groups:

	−20° to −10°	−10° to +10°	+10° to +20°
A	+3′·2	+5′·8	+13′·3
B	−1·3	−13·5	−8·7

The conclusion (*c*) was not altogether unexpected, for an inspection of Carrington's material had indicated something of the kind. The material is given on pp. 240–242 of his volume *Observations of Solar Spots* (published in 1863 by Williams & Norgate); and if it is divided into four groups according to the latitude of the spots, we get approximately for his X and Y—

Latitudes.	X	Y
> +15°	− 3′·8	+12′·9
+15° to 0°	− 9·4	+10·2
0° to −15°	− 4·1	−15·4
> −15°	−15·0	+ 9·0
Mean	− 8·1	+ 4·2

It thus seems doubtful what precise meaning can be attached to the mean of such discordant values. On forming groups corresponding to those used in this paper there are still clear discordances; but the whole qnestion must be reserved for further examination.

Nothing has yet been said of the constant C indicated by the mean values of the columns. It will be seen that it is persistently negative, the mean value from Table I. being $-0^\circ{\cdot}25$, and from Table II. $-0^\circ{\cdot}13$. These values are consistent, and indicate a steady drift of $-0^\circ{\cdot}019$ or $-1'{\cdot}14$ for every 10° of advance across the disc. Carrington found a phenomenon of this kind, and thought its reality so improbable that he set it down to some error in reduction. His value was $1'{\cdot}5$ for 30° of rotation, or $18'$ for a whole rotation; and his direction was *towards* the North Pole, *i.e.* in the direction opposite to that here found. It thus becomes important to examine whether this general drift shows any clear indication of change during our period (1886–1901). For this purpose the material must be rearranged in consecutive years, as in Table IV.

TABLE IV.

Drift in Latitude between $+65^\circ$ and -65° from central meridian.

Year.	-25°	-20°	-15°	-10°	0°	$+10^\circ$	$+15^\circ$	$+20^\circ$	$+25^\circ$	
1886	—	—	-4_{9}	-13_{7}	$+8_{14}$	-14_{3}	$+27_{2}$	-57_{2}	—	—
1887	—	—	—	-70_{1}	$+29_{10}$	-4_{4}	-65_{3}	—	—	—
1888	—	—	—	-128_{2}	-102_{6}	$+110_{1}$	-145_{1}	—	—	—
1889	-2_{2}	$+30_{1}$	-60_{1}	—	-66_{5}	$+25_{1}$	—	—	—	—
1890	—	-68_{1}	—	—	-21_{2}	—	—	$+65_{1}$	-33_{1}	$+60_{1}$
1891	$+51_{2}$	$+98_{4}$	-27_{9}	$+35_{1}$	—	—	$+19_{6}$	-12_{17}	-41_{13}	$+42_{5}$
1892	$+2_{10}$	-114_{15}	-30_{13}	-26_{9}	$+57_{3}$	-35_{9}	-55_{26}	$+6_{10}$	$+5_{7}$	$+119_{5}$
1893	$+61_{4}$	-43_{11}	-47_{20}	-24_{26}	$+21_{23}$	-16_{10}	-19_{15}	-4_{17}	-83_{8}	-25_{1}
1894	$+47_{5}$	-25_{1}	-40_{16}	-50_{20}	$+3_{17}$	-11_{22}	-65_{18}	-7_{15}	$+4_{4}$	—
1895	—	$+63_{4}$	-49_{10}	-50_{18}	-31_{14}	-68_{5}	-48_{25}	-37_{14}	-26_{4}	—
1896	—	-42_{5}	-4_{10}	-16_{16}	$+16_{4}$	-95_{5}	-127_{3}	-122_{5}	-15_{1}	—
1897	—	—	-5_{2}	$+33_{6}$	-24_{19}	-77_{14}	-79_{5}	—	—	—
1898	—	—	—	$+3_{9}$	-26_{11}	-109_{6}	-100_{6}	—	—	—
1899	—	—	—	$+20_{7}$	$+40_{5}$	-23_{2}	—	—	—	—
1900	—	—	—	$+60_{1}$	-22_{6}	$+5_{2}$	—	—	—	—
1901	—	$+117_{1}$	—	—	—	$+17_{2}$	—	—	—	—

The unit in the table is 0°·01: the subscript figures represent the number of spots included in the mean.

The mean values of the drift, weighting all spots equally, are as follows:—

Table V.

Year.	Mean Drift.	No. Spots.	Year.	Mean Drift.	No. Spots.
1886	−0°·04	37	1894	− °·24	118
1887	·00	18	1895	− ·40	94
1888	− ·90	10	1896	− ·39	49
1889	− ·34	10	1897	− ·39	44
1890	− ·03	6	1898	− ·47	32
1891	− ·02	57	1899	+ ·21	14
1892	− ·30	107	1900	− ·07	9
1893	− ·19	135	1901	+ ·50	3

There are some indications of change, but the small number of observations in the terminal years makes it difficult to pronounce with certainty. The reduction of the material recently made available should go far to settle the point. Meanwhile we may sum up the indications by taking means for four years as follows, Carrington's result being rendered comparable and added:—

Mean drift during a whole rotation.

1853–1861 Carrington.		1886–1889	1890–1893	1894–1897	1898–1901
+18′	* * *	−30′	−32′	−56′	−30′

[Note, added December 28.—A provisional reduction of the results for 1874–1885 shows that the drift is distinctly *positive* in those years, being smaller at the ends and large in the middle. An oscillation of about 26 years' period with maximum northerly drift about 1854, 1880, 1906, and maximum southerly about 1867, 1893, would roughly fit the facts observed hitherto.]